



# PROJECT-SET

Statistics Education for Teachers

## Open Ended Task: Valentine Marbles

### Grading Rubric

Task adopted from [www.illustrativemathematics.org](http://www.illustrativemathematics.org) aligned with standard 7.SP.2

Written task aligned with SV: Loop 1

**Grade each question below using the following three grades: Essentially Correct (E), Partially Correct (P), and Incorrect (I).**

**Example answers for each category are given below each question.**

**Once each of the questions is graded in this manner, the results will then be tallied to give an overall score for the task. For each E, a person will receive 1 point. For each P, a person will receive  $\frac{1}{2}$  point. For each I, a person will receive 0 points. The overall grade will be one of the following:**

**5 Complete Response**

**3.5/4.5 Substantial Response**

**3/2 Developing Response**

**1/1.5 Minimal Response**

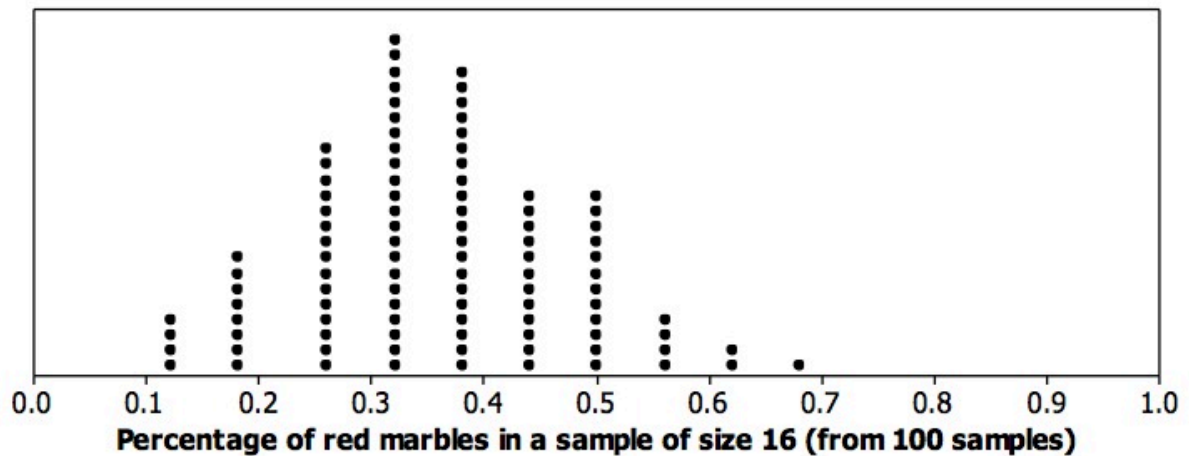
**When grading, please write the amount of points earned and the word to describe the points earned (complete, substantial, developing, or minimal) on the front page of each person's paper.**

A hotel holds a Valentine's Day contest where guests are invited to estimate the percentage of red marbles in a huge clear jar containing both red marbles and white marbles. There are 11,000 total marbles in the jar: 4,125 are red, 6,875 are white. The actual percentage of red marbles in the entire jar 37.5% is known to some members of the hotel staff.

Any guest who makes an estimate that is within 9 percentage points of the true percentage of red marbles in the jar wins a prize, so any estimate from 28.5% to 46.5% will be considered a winner. To help with the estimating, a guest is allowed to take a random sample of 16 marbles from the jar in order to come up with an estimate. (Note: when this occurs, the marbles are then returned to the jar after counting.)

One of the hotel employees who does not know that the true percentage of red marbles in the jar is 37.5% is asked to record the results of the first 100 random samples. A table and dotplot of the results appears below.

Percentage of red marbles in the sample of size 16	Number of times the percentage was obtained
12.50%	4
18.75%	8
25.00%	15
31.25%	22
37.50%	20
43.75%	12
50.00%	12
56.25%	4
62.50%	2
68.75%	1
<b>Total:</b>	100



- Assuming that each of the 100 guests who took a random sample used their random sample's red marble percentage to estimate the whole jar's red marble percentage. Based on the table above, circle the winners on the dot plot. How many of these guests would be "winners"? Explain your answer.

Grading Category	Solutions, Explanations, and Sample Answers
Essentially Correct (E)	Since any estimate from 28.5% to 46.5% will be considered a winner, all of the estimates of 31.25%, 37.5% and 43.75% would be the "winners." From the table or dot plot, that would be 22 + 20 + 12 respectively, which would make for a total of 54 winning estimates out of 100 or 54% of the sample estimates.

	Note: if the person circled the correct dots on the dot plot but made a counting error, they should still be given an E.
Partially Correct (P)	A partially correct answer would either only make note of the potential values that would be winners without counting those for this example or it would only give the number 54 without explanation.  Sample Answer: The winners would be those who used between 28.5% to 46.5% as their estimate.
Incorrect (I)	An incorrect answer would fail to meet the criteria of E and P.  Sample Answer: The winners would estimate 37.5%.

2. How many of the 100 guests obtained a sample that was *more than* half red marbles? Explain your answer.

<b>Grading Category</b>	<b>Sample Answer</b>
Essentially Correct (E)	From the table or dot plot, an estimate that is "more than half" would be any estimate of 56.25%, 62.5%, or 68.75%. That would be 4 + 2 + 1 respectively, which would make for a total of 7 estimates out of 100 that have values that correspond to "more than half red."
Partially Correct (P)	A partially correct answer would either state that the estimates with more than 50% were 56.25%, 62.5%, or 68.75% or that those larger than 50% should be counted.  Sample Answer: The guests that obtained more than half the red marbles would have an estimate above 50%.
Incorrect (I)	An incorrect answer is one that fails to meet the criteria of E or P.  Sample Answer: There were 12 guests that estimated 50%.

3. Explain what this table and dot plot represents. In your own words, what does the table and the dot plot above illustrate? How do the table and the dot plot illustrate the sample-to-sample variation of the proportion of red marbles picked?

<b>Grading Category</b>	<b>Sample Answer</b>
Essentially Correct (E)	<p>The table and dot plot illustrate the variation of the proportion of red marbles from the samples chosen by the guests. Assuming the samples were randomly selected, then the table and dot plot illustrate an approximate sampling distribution for the sample proportion of red marbles.</p> <p>The table and dot plot illustrate that the sample proportions by using a dot on the plot and an entry in the table to represents each one of the sample proportions found by the hotel guests.</p>
Partially Correct (P)	A partially correct answer would either answer the first question only or the second question correctly.
Incorrect (I)	An incorrect answer is one that fails to meet the criteria of E or P.

4. Should we be concerned that not all of the samples had a red marble percentage of exactly 37.5.% even though that value is the true red marble percentage for the whole jar? Explain your answer.

<b>Grading Category</b>	<b>Sample Answer</b>
Essentially Correct (E)	<p>No, we should not be concerned due to sampling variability. It is very plausible that the true population proportion is never sampled. However, we are likely to see that percentages close to the true population proportion are sampled with a higher likelihood than those that are very different. If we did not, we may be concerned that the sampling was not done in a random manner. In this case, we see that values between 0.3 and 0.4 are often sampled thus there should be no concern.</p> <p>There may also be mention of the sample size and how as it increases then the closeness of the sample proportion to the population proportion should improve. Discussion about the sample size ONLY is not acceptable for an E.</p>
Partially Correct (P)	A partially correct answer is one that either (1) demonstrates correct reasoning but has some mathematical misstatement, (2) discusses the sample size correctly but does not mention variability, or (3) makes only general statements

	<p>about being close to 37.5%.</p> <p>Sample Answer: No, there were lots of answers close to 37.5%.</p>
Incorrect (I)	<p>An incorrect answer is one that fails to meet the criteria of E or P.</p> <p>Sample Answer: Yes, we should be concerned. Since we would expect to obtain 37.5% most of the time.</p>

5. Recall that the hotel employee who made the table and dot plot above didn't know that the real percentage of red marbles in the entire jar was 37.5%. If another person thought that half of the marbles in the jar were red, explain briefly how the hotel employee could use the dot plot and table results to challenge this person's claim. Specifically, what aspects of the table and dot plot would encourage the employee to challenge the claim?

Grading Category	Sample Answer
Essentially Correct (E)	<p>Under the assumption that the random samples are representative of the population from which they were selected, if half of the marbles in the glass jar were red, then the dot plot and table would most likely show a majority of sample values centered near 50%. We notice from the collected data that most of the sample estimates are in the 25% to 43.75% range, and the graph seems to be centered more in the low-30% values.</p>
Partially Correct (P)	<p>A partially correct answer would be one that mentions that the center of the values is not at 50%. Discussion of how the graph is centered at a value far from 50% and/or how a small number of the estimates are actually near 50% would also be appropriate.</p> <p>Other partially correct answers may include arguments that take the wrong perspective. Arguments that assume the hotel employee knows the true percent of 33.6% but make correct observations should be awarded a P.</p> <p>Sample Answer: There are more estimates of the number of reds around 30% or 40% compared to 50% reds.</p>
Incorrect (I)	<p>An incorrect answer is one that fails to meet the criteria for E or P.</p> <p>Sample Answer: The hotel employee doesn't need to challenge since a few of the estimates are 50% or more.</p>